

Coupons Cannot Certify Components: A Weakest-Link Analysis of Defect-Driven Strength and Design Allowables in Additively Manufactured Metal

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ABSTRACT

Strength in additively manufactured metal is usually governed by the largest process-induced defect in the stressed volume rather than by bulk microstructure. Design allowables, however, are derived from small test coupons by statistical procedures that assume the coupon and the component are drawn from the same strength distribution. Under weakest-link behaviour they are not, because a larger component samples a larger defect population and therefore contains a larger worst defect. This study quantifies the volumetric size effect implied by a defect-driven strength model, determines how far coupon-derived A-basis and B-basis allowables mispredict component strength, and compares two process-improvement routes on a common basis. A weakest-link Monte Carlo model was constructed. Defects were distributed as a Poisson process at 0.8 per mm³ with $\sqrt{\text{area}}$ log-normally distributed about a median of 45 μm . Strength was computed from the governing defect by a Murakami-type relation for a material of 380 HV. Stressed volumes from 10 to 100,000 mm³ were simulated with 20,000 realisations each. A-basis and B-basis allowables were computed from simulated coupon sets of 5 to 100 specimens of 10 mm³ and compared with the strength distribution of a 100,000 mm³ component. Two mitigations were compared: reducing median defect size from 45 to 20 μm , and reducing defect density from 0.8 to 0.08 per mm³. Mean strength fell from 334.9 MPa at 10 mm³ to 254.8 MPa at 100,000 mm³, a reduction of 23.9 % across four decades of volume, following a power law with exponent -0.0292 . The coefficient of variation fell simultaneously from 6.13 % to 2.51 %, so larger components were both weaker and more consistent. B-basis allowables derived from 10 mm³ coupons ranged from 268.2 MPa at five specimens to 296.8 MPa at one hundred, while the 10th percentile of the 100,000 mm³ component was 246.5 MPa; at every coupon count tested, essentially the entire component population fell below the coupon-derived B-basis. Testing more coupons made this worse, because a tighter estimate of the coupon distribution raises the allowable toward a mean that is itself wrong for the component. Halving the median defect size raised mean strength by 41.0 MPa; a tenfold reduction in defect density raised it by 20.5 MPa. Where strength is defect-limited, coupon-derived allowables are non-conservative for larger components by construction, and increasing the coupon count does not correct the error. Allowables should be scaled to the stressed volume by an explicit size-effect law, and process improvement should target the extreme of the defect size distribution rather than the mean defect count.

Keywords: additive manufacturing; weakest-link statistics; defect-driven failure; design allowables; size effect; extreme value statistics.

1. INTRODUCTION

Metal additive manufacturing has moved from prototyping into load-bearing structure, and with it has come a qualification problem that conventional wrought practice does not prepare engineers for [8-10]. In a wrought alloy, strength is a property of microstructure, coupons and components share that microstructure, and the statistical machinery of design allowables — test a set of coupons, apply a tolerance factor, publish an A-basis or B-basis number — is sound because both are samples from the same population [17,18].

In an additively manufactured part, strength is frequently governed by the largest lack-of-fusion pore or unmelted region in the stressed volume [6,7,11,12]. Failure initiates there, and the bulk microstructure sets only the resistance to a crack once it has started [14]. That changes the statistics fundamentally: the property being measured is not a mean over the material but an extreme over a defect population, and extremes depend on how much material is searched.

The consequence is a size effect [3,15]. A 10 mm³ coupon contains a handful of defects and its worst one is modest; a 100,000 mm³ component contains ten thousand times as many and its worst one is much larger. If strength falls with the size of the governing defect, the component is systematically weaker than the coupon, and no amount of coupon testing will reveal it because the coupons never contain a defect of the size the component will.

This study quantifies that effect. Three questions are addressed: how large the volumetric size effect is for a representative defect population; how far coupon-derived allowables mispredict component strength and whether testing more coupons helps; and which of two process-improvement routes — shrinking defects or reducing their number — buys more strength per unit of effort.

2. MODEL

2.1 Weakest-Link Formulation

Figure 1 shows the chain from process parameters to component strength.

Defects are treated as a homogeneous Poisson process in the stressed volume with intensity λ per unit volume, so a component of volume V contains a number of defects distributed as $\text{Poisson}(\lambda V)$. Each defect is characterised by the square root of its projected area, the standard length parameter for defect-initiated failure [1,2], drawn from a log-normal distribution.

The component fails at its worst defect, so the governing defect size is the maximum of the sampled set [3,16], and strength follows from a Murakami-type relation in which the critical stress scales with the inverse sixth root of the defect size [1,2]:

$$\sigma_c = \frac{C(HV+120)}{(\sqrt{\text{area}})^{1/6}} \quad (1)$$

with $C = 1.43$ for internal defects and $\sqrt{\text{area}}$ in micrometres. The combination of a Poisson defect count, a log-normal size distribution and an extreme-value maximum is what generates the size effect; the Murakami relation converts defect size into strength but contributes no volume dependence of its own. Defect-based models of this kind have been used to predict the strength of additively manufactured alloys [4,5]. The model parameters are listed in Table 1.

The chain from process to strength

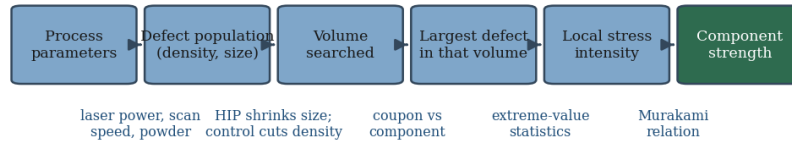


Fig.1: The chain from process parameters to component strength.

2.2 Design Allowables

A-basis and B-basis allowables were computed from simulated coupon sets by the conventional one-sided tolerance-limit procedure, subtracting a tolerance factor times the sample standard deviation from the sample mean [17,18]. B-basis is intended to bound the tenth percentile with 95 % confidence and A-basis the first percentile with 95 % confidence, both computed on the assumption that the coupons and the structure share a distribution.

Coupons were modelled as 10 mm³ of stressed material and the component as 100,000 mm³. Four hundred independent coupon sets were simulated at each sample size so that the reported allowables are averages over the sampling variation an engineer would experience, rather than a single draw.

Table 1: Model parameters.

Parameter	Value	Role
Vickers hardness, HV	380	Sets the strength scale in Equation (1)
Murakami constant, C	1.43	Internal defect case
Defect density, λ	0.8 per mm ³	Baseline; varied to 0.08 in Section 3.4
Median $\sqrt{\text{area}}$	45 μm	Baseline; varied to 20 μm in Section 3.4
Log-normal shape, σ	0.55	Spread of the defect size distribution
Stressed volume	10 to 100,000 mm ³	Coupon to component
Realisations	20,000 per volume	Monte Carlo error negligible

Note: Parameter values are representative of a defect-containing laser powder-bed fusion alloy and are stated assumptions rather than measurements from any specific build. Absolute strengths are therefore illustrative; the volumetric scaling and the allowables result follow from the structure of the model.

3. RESULTS

3.1 The Size Effect

Table 2 and Figure 2 give strength and governing defect size as a function of stressed volume.

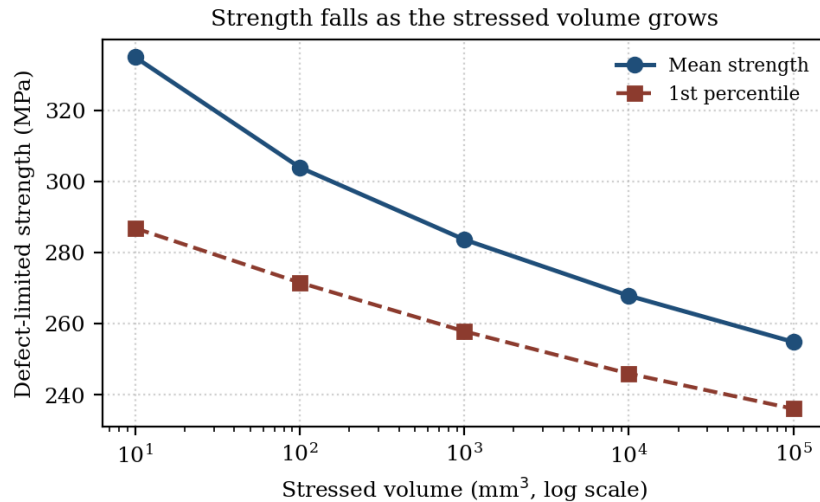


Fig.2: Mean and first-percentile strength against stressed volume.

Mean strength falls from 334.9 MPa to 254.8 MPa across four decades of volume, a reduction of 23.9 %. A power-law fit gives an exponent of -0.0292 , so strength scales approximately as V raised to the power -0.029 — slow, but relentless, and large in aggregate over the volume ratios that separate a coupon from a real part.

The mechanism is visible in the final two columns of Table 2 and in Figure 3. The median governing defect grows from 94.7 μm to 478.3 μm , a factor of five, simply because a larger volume searches further into the tail of the same size distribution. Nothing about the material has changed; only the amount of it that is being asked to survive.

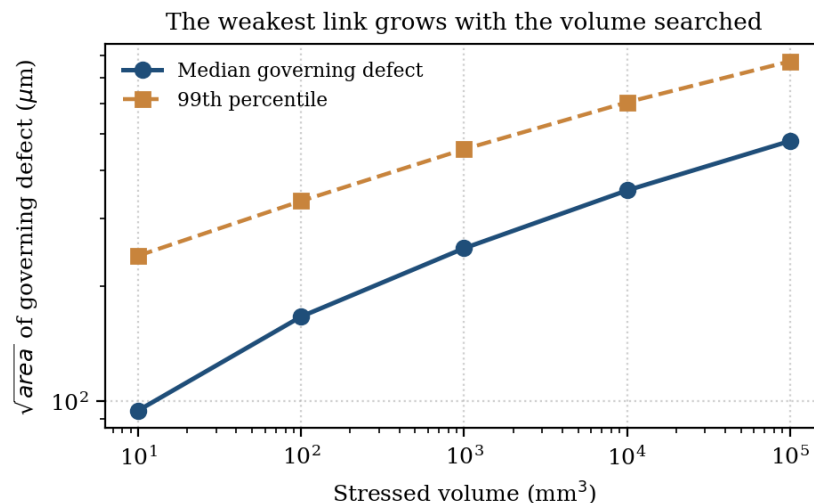


Fig.3: Size of the governing defect against stressed volume, on logarithmic axes.

3.2 Scatter Narrows as Strength Falls

As Figure 4 shows, the coefficient of variation falls from 6.13 % at 10 mm³ to 2.51 % at 100,000 mm³. Larger components are simultaneously weaker and more consistent, which is a genuine prediction of extreme-value behaviour and runs against the intuition that more material means more variability [16].

Table 2: Strength and governing defect size by stressed volume.

Volume (mm ³)	Mean strength (MPa)	SD (MPa)	CV (%)	1st percentile (MPa)	Median governing defect (μm)	99th percentile defect (μm)
10	334.9	20.51	6.13	286.8	94.7	240.1
100	303.9	12.12	3.99	271.5	166.5	333.6
1,000	283.6	9.20	3.24	257.8	251.1	455.3
10,000	267.8	7.44	2.78	245.9	355.5	603.7
100,000	254.8	6.39	2.51	236.0	478.3	772.4

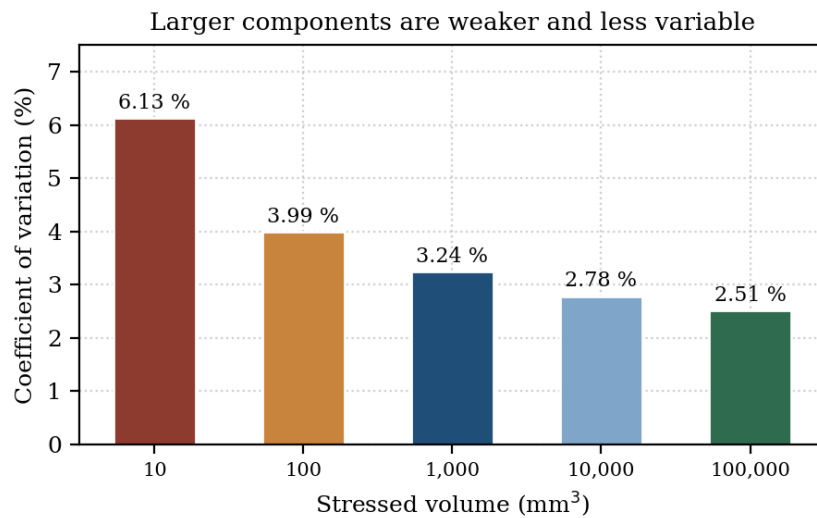


Fig.4: Coefficient of variation of strength against stressed volume.

The explanation is that a large volume almost always contains a defect from the far tail of the size distribution, so the maximum is reliably large; a small coupon sometimes draws a bad defect and sometimes does not, and it is that lottery which produces the coupon scatter. Coupon-level variability is therefore not a measure of material inconsistency so much as a measure of how unlikely a small volume is to contain the worst defect the process can produce.

This has an uncomfortable corollary for qualification. A programme that judges process stability by the scatter of coupon results will read a tightening of that scatter as an improvement, when the same tightening occurs simply by testing larger coupons. Scatter should be interpreted alongside the volume it was measured in, and reported with it.

3.3 Design Allowables

Table 3 and Figure 5 compare coupon-derived allowables with the strength distribution of the 100,000 mm³ component.

Table 3: Coupon-derived allowables against the component distribution.

Coupons tested	B-basis from coupons (MPa)	A-basis from coupons (MPa)	Component 10th percentile (MPa)	Component 1st percentile (MPa)	Component population below the B-basis (%)
5	268.2	222.4	246.5	236.4	99.81
10	282.3	248.2	246.5	236.4	100.00
20	290.7	262.6	246.5	236.4	100.00

Coupons tested	B-basis from coupons (MPa)	A-basis from coupons (MPa)	Component 10th percentile (MPa)	Component 1st percentile (MPa)	Component population below the B-basis (%)
30	293.2	267.0	246.5	236.4	100.00
50	295.0	270.9	246.5	236.4	100.00
100	296.8	274.0	246.5	236.4	100.00

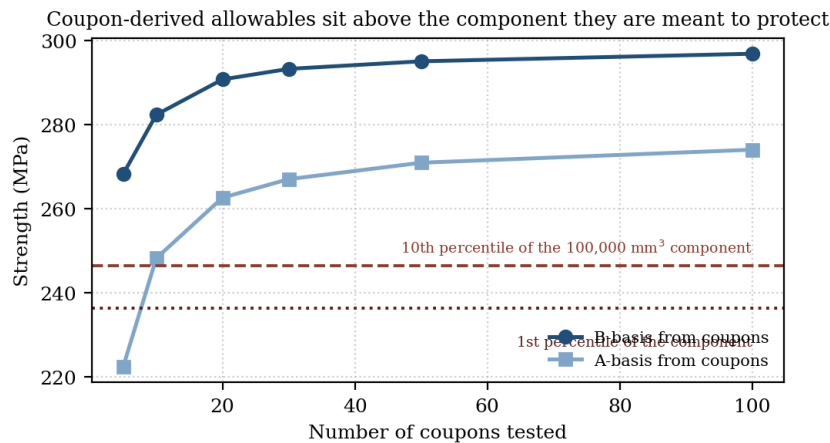


Fig.5: Coupon-derived allowables against the strength distribution of the 100,000 mm³ component.

This is the central result. A B-basis allowable is intended to be a strength that 90 % of the structure exceeds, with 95 % confidence. Derived from 10 mm³ coupons it lies between 268.2 and 296.8 MPa depending on how many were tested, while the 10th percentile of the 100,000 mm³ component is 246.5 MPa. Essentially the entire component population falls below the number nominally protecting it.

The A-basis figures are worse than they look. At five coupons the A-basis of 222.4 MPa does lie below the component distribution, but only because a five-specimen tolerance factor is so large that it produces a conservative number by accident. As the coupon count rises the tolerance factor shrinks, the A-basis climbs toward the coupon mean, and by 100 coupons it reaches 274.0 MPa — above even the component tenth percentile.

Testing more coupons makes the problem worse. This is the finding most likely to be counterintuitive to a qualification engineer, and it is not a subtlety of the model. Tolerance factors exist to account for uncertainty in estimating the coupon distribution; they shrink as the sample grows, because that uncertainty shrinks [18]. They do nothing about the fact that the coupon distribution is the wrong distribution. Additional testing therefore purchases precision about a quantity that does not describe the component, and the resulting allowable converges upward onto a mean that is roughly 80 MPa too high.

No statistical treatment of coupon data can repair this, because the information required — the strength of a volume ten thousand times larger — is not present in the data. It has to come from a model of the size effect, from component-scale testing, or from an explicit volumetric knock-down factor.

3.4 Two Routes to a Stronger Part

Table 4 and Figure 6 compare two process-improvement routes and their combination at 1,000 mm³.

Table 4: Effect of two process improvements at 1,000 mm³.

Configuration	Mean strength (MPa)	CV (%)	1st percentile (MPa)	Gain in mean (MPa)	Gain at 1st percentile (MPa)
Baseline: 45 μ m median, 0.8 defects/mm ³	283.4	3.25	257.6	—	—
Hot isostatic pressing: median 45 $\rightarrow$ 20 μ m	324.4	3.25	294.9	+41.0	+37.4
Process control: density 0.8 $\rightarrow$ 0.08 /mm ³	303.9	3.96	271.4	+20.5	+13.9
Both	347.8	3.96	310.6	+64.4	+53.2

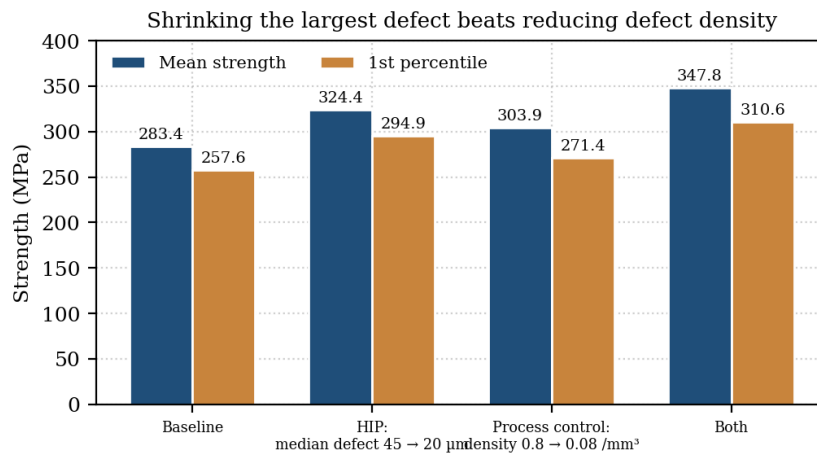


Fig.6: Strength under two process-improvement routes and their combination.

Reducing the median defect size by a factor of 2.25 gained 41.0 MPa in the mean and 37.4 MPa at the first percentile. A tenfold reduction in defect density gained 20.5 MPa and 13.9 MPa respectively — half as much for what is, in process terms, the more demanding intervention.

The asymmetry follows directly from the model structure. Strength depends on the size of the largest defect, and the largest defect depends on the size distribution through its position and on the density only through how far into the tail the sampling reaches. Reducing density by a factor of ten moves the expected maximum back by roughly one decade of the tail; reducing the whole distribution by a factor of 2.25 moves every defect, including the largest.

The two are not equivalent in their effect on scatter either. Reducing defect size left the coefficient of variation unchanged at 3.25 %, while reducing density raised it from 3.25 % to 3.96 %, because a sparser defect population makes the maximum more variable [13]. A process change that improves mean strength while increasing scatter can reduce the allowable, and in this case the density route delivered less than half the first-percentile gain of the size route despite a respectable gain in the mean.

4. DISCUSSION

4.1 Principal Findings

Mean defect-limited strength fell 23.9 % across four decades of stressed volume, following a power law with exponent -0.0292 , while the coefficient of variation more than halved. B-basis allowables derived from 10 mm³ coupons exceeded the tenth percentile of a

100,000 mm³ component by 22 to 50 MPa, and the discrepancy widened as more coupons were tested. Reducing the median defect size delivered twice the strength gain of a tenfold reduction in defect density, and did so without the increase in scatter that the density route produced.

4.2 Implications for Design and Qualification

Table 5 summarises each finding, its implication and the corresponding recommendation.

Table 5: Implications.

Finding	Implication	Recommendation
Strength scales as $V^{-0.03}$	Coupon strength overstates component strength	Apply an explicit volumetric knock-down, fitted to the local defect population
Coupon B-basis exceeded component 10th percentile	Allowables are non-conservative by construction	Do not certify defect-limited parts on coupon allowables alone
More coupons raised the allowable	Extra testing buys precision about the wrong distribution	Increase coupon volume or test at component scale, not coupon count
CV fell with volume	Low coupon scatter is not evidence of process stability	Report scatter with the volume it was measured in
Defect size beat defect density 2:1	The extreme, not the average, governs	Prioritise hot isostatic pressing and defect-closing steps
Density reduction raised scatter	A gain in the mean can be lost in the allowable	Evaluate process changes on the design percentile, not the mean
The governing defect grew fivefold with volume	Inspection sensitivity must match the part, not the coupon	Set non-destructive inspection thresholds from the component volume

The third row is the one with the clearest cost implication. A qualification programme responding to uncertainty by testing more coupons is spending money to move its allowable in the wrong direction. The same budget spent on fewer, larger specimens — or on a small number of component-scale tests — addresses the actual source of error, because it samples the defect population at a scale closer to the part.

The final row connects the statistics to inspection. If the governing defect in a 100,000 mm³ component has a median $\sqrt{\text{area}}$ of 478 μm and a 99th percentile of 772 μm , then an inspection technique qualified to detect 250 μm defects in a coupon is being asked to find something quite different in the part, and its probability of detection should be established at the relevant size [9,10].

4.3 Limitations

The model treats defects as a homogeneous Poisson process, which real builds are not. Lack-of-fusion porosity clusters at layer boundaries, at scan-vector turnarounds and in regions of unstable melt pool behaviour, and it varies systematically with build height and with proximity to overhangs and supports [11,13]. Clustering makes the effective volume searched smaller than the geometric volume and would reduce the size effect somewhat; systematic variation with position would introduce a dependence on where in the build the coupon was taken that the model does not represent at all.

Strength is computed from a Murakami-type relation developed for fatigue limits in the presence of small defects, applied here as a general defect-limited strength criterion [1,2]. The inverse sixth-root dependence is characteristic of that formulation and other criteria give different exponents; a weaker dependence on defect size would reduce the size effect

proportionately. Stress state is treated as uniform over the stressed volume, whereas real components have gradients, so the effective volume at risk is smaller than the geometric one — a well-known correction that would shift the absolute numbers without removing the effect.

Residual stress, surface roughness and build orientation are omitted entirely, and in as-built surfaces the governing defect is frequently at or near the surface rather than internal, where the appropriate Murakami constant differs. Anisotropy is not modelled. All parameter values are representative assumptions rather than measurements, so absolute strengths should not be quoted for any alloy or process.

Finally, the allowables analysis assumes the conventional normal-theory tolerance-limit procedure. Practitioners aware of the weakest-link problem sometimes fit Weibull distributions and scale by volume explicitly [3,15], which is precisely the remedy recommended here; the analysis demonstrates the consequence of not doing so rather than criticising those who already do.

5. CONCLUSION

When strength is set by the largest defect in the stressed volume, it is a property of the volume as much as of the material. In this model mean strength fell from 334.9 MPa in a 10 mm³ coupon to 254.8 MPa in a 100,000 mm³ component, the governing defect grew from 95 to 478 µm, and the scatter halved — so the component was weaker, more predictable, and entirely below the B-basis allowable that coupon testing produced for it.

Two practical conclusions follow. Coupon-derived allowables cannot certify a defect-limited component, and testing more coupons moves the allowable further from the truth rather than closer to it; the missing information is a size-effect law, and it must come from larger specimens, component-scale tests or an explicit volumetric correction. And where process improvement is the objective, shrinking the largest defects delivered twice the strength gain of reducing how many there are, while leaving scatter unchanged — because in a weakest-link material it is the extreme of the distribution that is loaded, not its average.

DECLARATIONS

Ethical approval. Not applicable. This is a computational study; no human or animal subjects were involved.

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Conflicts of interest. The author declares no conflicts of interest, financial or non-financial, in relation to the subject matter of this manuscript.

Materials disclaimer. No specific alloy, machine, powder or commercial process is evaluated. Parameter values in Table 1 are representative assumptions and absolute strengths are illustrative; they must not be used as design data for any material.

Data and code availability. This is a computational study; no experimental or external datasets were used. All results were generated by the Monte Carlo model described in Section 2, run with a fixed random seed so that every reported value can be reproduced exactly from the parameters listed in Table 1. The code is available from the corresponding author upon reasonable request.

Author's contribution. Dr. Mohammed Hasan Ali is the sole author of this work and was responsible for the conception and design of the study, development of the weakest-link model, the Monte Carlo simulations and allowables analysis, interpretation of the results, preparation of the figures and tables, and drafting, revision and final approval of the manuscript. The author is accountable for all aspects of the work.

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