

The Air Matters More Than the Fins: Operating-Point Dependence and Optimum Flatness in Air-Cooled Heat Sink Design

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ABSTRACT

Fin spacing is the variable most often optimised in air-cooled heat sink design, and the optimisation is frequently performed at a fixed approach velocity. Under a fixed velocity the flow rate is an output; under a fixed fan or pumping power it is a constraint that the design itself changes, and the two formulations do not have the same optimum. This study determines how the optimum fin spacing shifts between these formulations, how much thermal performance is lost by optimising under the wrong one, how broad the optimum is, and how the total resistance divides between convection and the heating of the air. A parallel-plate heat sink of 80 mm base width, 80 mm flow length and 40 mm fin height in aluminium was modelled with standard correlations: developing laminar duct flow between fins with an aspect-ratio-corrected thermal entry Nusselt number, a developing-flow friction factor, and straight-fin efficiency. Total thermal resistance was taken as the sum of a convective term and a caloric term representing the temperature rise of the air. Fin count was swept from 4 to 60 at a fixed velocity of 3 m/s, at fixed pumping powers of 0.10, 0.50 and 2.00 W, and against a linear fan curve of 60 Pa shut-off and $0.02 \text{ m}^3\text{s}^{-1}$ free delivery. The optimum spacing shifted from 1.39 mm at 0.10 W to 0.84 mm at 2.00 W, with the optimum fin count rising from 34 to 44. Designing at the fixed-velocity optimum of 37 fins cost 2.0 %, 0.6 % and 6.7 % of thermal resistance at the three power budgets. Against that, raising the pumping power from 0.10 to 2.00 W reduced minimum resistance from 0.2204 to 0.1033 K/W, a 53 % improvement — roughly eight times the largest penalty from choosing the wrong spacing. The optimum was broad: at 0.50 W, every spacing between 0.84 and 1.39 mm fell within 5 % of the minimum. At all operating points the caloric term dominated, contributing 58.8 to 64.7 % of total resistance. Fin spacing is a forgiving design variable and the air path is not. Effort spent optimising spacing to the nearest tenth of a millimetre buys single-digit percentages, while the pumping power available buys tens of percent, and at every operating point examined the dominant resistance was heating the air rather than getting heat into it.

Keywords: heat sink design; fin spacing optimisation; thermal resistance; pumping power; electronics cooling; systems design.

1. INTRODUCTION

Air-cooled plate-fin heat sinks remain the default thermal solution for electronics, and fin spacing is the parameter their design most often turns on [3,7,14]. The trade is familiar: closer fins give more surface area but narrower channels, which raises the pressure drop and,

for a given fan, reduces the flow. Somewhere between a few large channels and many small ones there is an optimum [5,6,8].

Where that optimum lies depends on a modelling choice that is made early and often implicitly. If the analysis fixes the approach velocity and varies the geometry, flow rate follows from the geometry and the pressure drop is simply an output to be reported. If instead the fan or the pumping power is fixed, the flow rate is determined by the intersection of the sink's hydraulic resistance with the supply, so changing the fin spacing changes the flow through the sink. These are different problems and they do not share a solution.

The second formulation is the one that describes a real product, where a fan of known characteristic is fitted and delivers whatever flow the system resistance permits. The first is easier and is common in parametric studies and in the fin-spacing correlations quoted in design handbooks [4,9].

This study quantifies the consequences. Four questions are addressed: how far the optimum spacing moves between the formulations and across operating points; what it costs to optimise under the wrong one; how sharp the optimum is, and therefore how much precision in fin spacing is worth; and how the total thermal resistance divides between the convective and caloric contributions, which determines where further effort should go.

2. MODEL

2.1 Geometry and Flow

A parallel-plate sink of base width W , flow length L and fin height H with N fins of thickness t was modelled. Channel spacing follows from the fin count as $s = (W - Nt)/(N - 1)$, and the hydraulic diameter of each rectangular channel is $D_h = 2sH/(s + H)$.

Channel Reynolds number is computed from the mean channel velocity. In the laminar regime the thermal entry length is significant at these dimensions, so an aspect-ratio-corrected developing-flow Nusselt number was used [1,2,4],

$$Nu = 7.54 + \frac{0.03 \frac{1}{L^*}}{1 + 0.016 \left(\frac{1}{L^*} \right)^{2/3}} \quad (1)$$

where $L^* = L/(D_h Re Pr)$ is the dimensionless thermal entry length, with a corresponding developing-flow friction factor. Above a channel Reynolds number of 2300 the Dittus–Boelter and Blasius relations were used instead [3]. Fin efficiency follows the standard straight-fin result [3,14]

$$\eta = \frac{\tanh(mH)}{mH}, m = \sqrt{\frac{2h}{k_s t}} \quad (2)$$

2.2 Thermal Resistance

Total resistance from the base to the incoming air is taken as the sum of two terms [7,9]:

$$R = \frac{1}{h A_{eff}} + \frac{1}{\rho V \dot{c}_p} \quad (3)$$

The first is the convective resistance over the fin and base area, with fin area weighted by efficiency. The second is the caloric resistance: even with an infinitely effective surface, the air leaving the sink has been heated, and that temperature rise appears as an irreducible resistance set only by the mass flow. Separating the two is the key to interpreting the results, because they respond to different design levers.

2.3 Operating-Point Formulations

Table 1 lists the formulations compared and Table 2 gives the geometry and properties of the modelled sink.

Table 1: The four formulations compared.

Formulation	What is held fixed	What the design changes	Represents
Fixed velocity	Channel velocity at 3 m/s	Flow rate rises with channel area	A parametric study or handbook correlation
Fixed pumping power	Fan power at 0.10, 0.50 or 2.00 W	Velocity adjusts to keep $\Delta p \times \dot{V}$ constant	A design constrained by a power or acoustic budget
Fan curve	Linear fan characteristic, 60 Pa shut-off, $0.02 \text{ m}^3\text{s}^{-1}$ free delivery	Operating point moves along the fan curve	A product with a specified fan fitted

Table 2: Geometry and properties.

Quantity	Value
Base width $\times$ flow length	80 mm $\times$ 80 mm
Fin height	40 mm
Fin thickness	1.0 mm
Sink conductivity	$200 \text{ W m}^{-1} \text{ K}^{-1}$
Dissipated power	100 W
Air density, viscosity, conductivity	1.10 kg m^{-3} , $1.95 \times 10^{-5} \text{ Pa s}$, $0.0276 \text{ W m}^{-1} \text{ K}^{-1}$
Prandtl number	0.712
Fin count swept	4 to 60

Note: Air properties are evaluated at approximately 320 K. Geometry is representative of a mid-size electronics heat sink; absolute resistances are specific to it and the comparisons between formulations are the transferable result.

3. RESULTS

3.1 The Optimum Moves with the Operating Point

Table 3 and Figure 1 give the optimum design under each formulation.

The optimum spacing narrows as the power budget grows, from 1.39 mm at 0.10 W to 0.84 mm at 2.00 W, with the fin count rising from 34 to 44. The direction is expected: more available pumping power pays for the higher pressure drop of narrower channels, so the balance moves toward surface area.

The fixed-velocity case sits in the middle of that range at 1.19 mm and 37 fins, which is why handbook spacing rules derived at a nominal velocity are usually serviceable [6,17]. It is serviceable rather than correct, and Section 3.2 quantifies the difference.

Table 3: Optimum design under each formulation.

Formulation	Optimum fins	Spacing (mm)	R (K/W)	Velocity (m/s)	Δp (Pa)	Power (W)	Re	Fin efficiency
Fixed velocity 3 m/s	37	1.19	0.2399	3.00	15.7	0.081	393	0.690
Fixed power 0.10 W	34	1.39	0.2204	3.44	15.8	0.100	522	0.715
Fixed power 0.50 W	39	1.08	0.1453	6.22	49.0	0.500	737	0.666
Fixed power 2.00 W	44	0.84	0.1033	10.33	134.5	2.000	955	0.615
Fan curve	37	1.19	0.1660	5.15	33.5	0.296	673	0.686

As Figure 2 shows, the fan-curve case is the most realistic and behaves differently from any fixed-power case, because the fan delivers more power into a low-resistance sink and less into a high-resistance one. Its optimum coincided with the fixed-velocity optimum at 37 fins, but at an operating point of 0.296 W — a power the designer did not choose and would not know without solving the intersection.

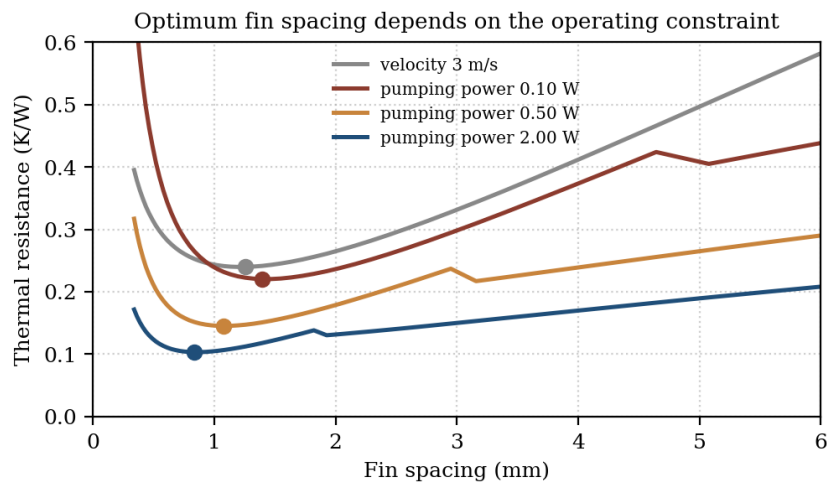


Fig.1: Thermal resistance against fin spacing under four formulations, with each optimum marked.

3.2 What Optimising under the Wrong Formulation Costs

Table 4 gives the penalty for adopting the fixed-velocity optimum at each power budget.

Table 4: Penalty for adopting the fixed-velocity optimum.

Power budget	Fins at velocity optimum	Fins at true optimum	R at velocity optimum (K/W)	R at true optimum (K/W)	Penalty
0.10 W	37	34	0.2248	0.2204	2.0 %
0.50 W	37	39	0.1462	0.1453	0.6 %
2.00 W	37	44	0.1102	0.1033	6.7 %

The penalty is small: between 0.6 % and 6.7 % of thermal resistance across a twentyfold range of pumping power. A designer who optimised at a nominal velocity and then fitted a fan delivering 2 W of air power would be leaving 6.7 % on the table, and less than that at lower powers.

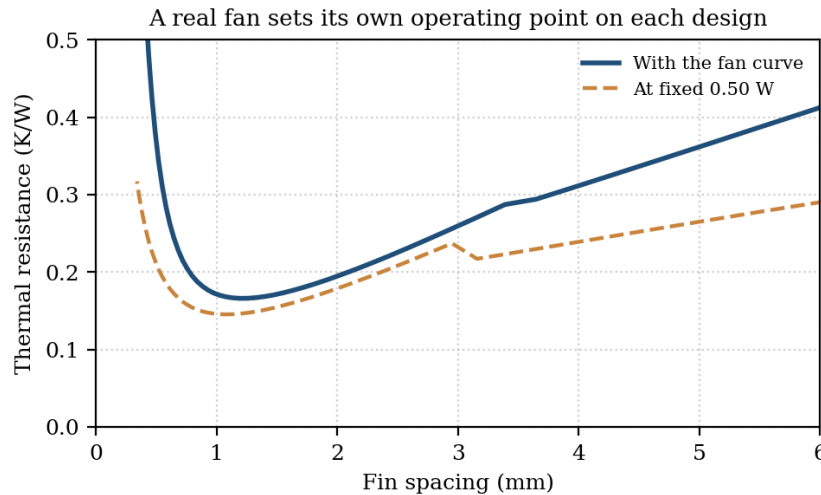


Fig.2: Resistance against spacing with a real fan curve compared with a fixed pumping power.

This is a negative result and it is worth reporting as one. The formulation does matter, the optimum does move, and the consequence of getting it wrong is smaller than the effort usually devoted to the question. Fin spacing optimisation is not where the performance is.

3.3 Where the Performance Actually Is

Figure 3 shows the optimum spacing and the minimum achievable resistance as the pumping power budget increases. Raising the pumping power from 0.10 to 2.00 W reduced minimum thermal resistance from 0.2204 to 0.1033 K/W, a 53 % improvement. Against a maximum spacing penalty of 6.7 %, the air budget is worth roughly eight times as much as the geometry decision it constrains.

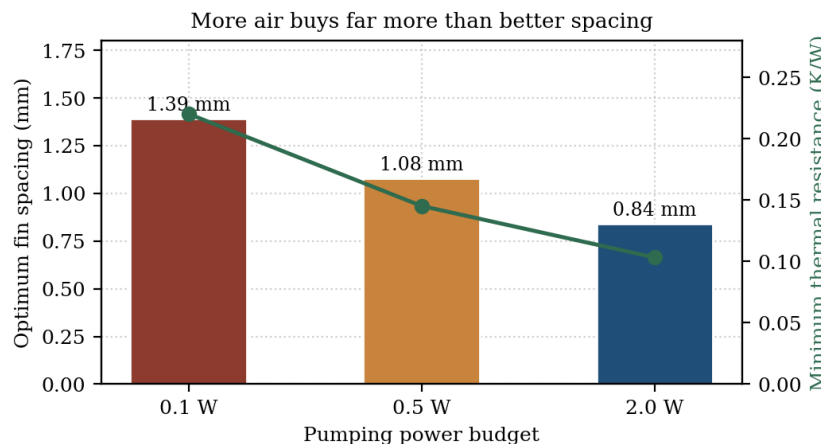


Fig.3: Optimum spacing and minimum achievable resistance against pumping power budget.

For the 100 W dissipation modelled, that is the difference between a base-to-air temperature rise of 22.0 K and 10.3 K. A designer facing a thermal shortfall has a choice between re-optimising the fin field, which will recover a few per cent, and finding another watt of fan power or reducing the system pressure drop upstream of the sink, which will recover tens of per cent. The second is almost always the better use of engineering effort, and it is frequently the harder conversation because it involves acoustics, enclosure design and someone else's subsystem.

3.4 How Sharp Is the Optimum?

Table 5 and Figure 4 give the range of fin spacing within 5 % of the minimum resistance under each formulation.

Table 5: Designs within 5 % of the minimum resistance.

Formulation	Optimum spacing (mm)	Spacing within 5 % of minimum	Fin count range	Number of designs
Fixed velocity 3 m/s	1.19	0.88 to 1.72 mm	30 to 43	14
Fixed power 0.10 W	1.39	1.14 to 1.82 mm	29 to 38	10
Fixed power 0.50 W	1.08	0.84 to 1.39 mm	34 to 44	11
Fixed power 2.00 W	0.84	0.65 to 1.14 mm	38 to 49	12
Fan curve	1.19	0.97 to 1.55 mm	32 to 41	10

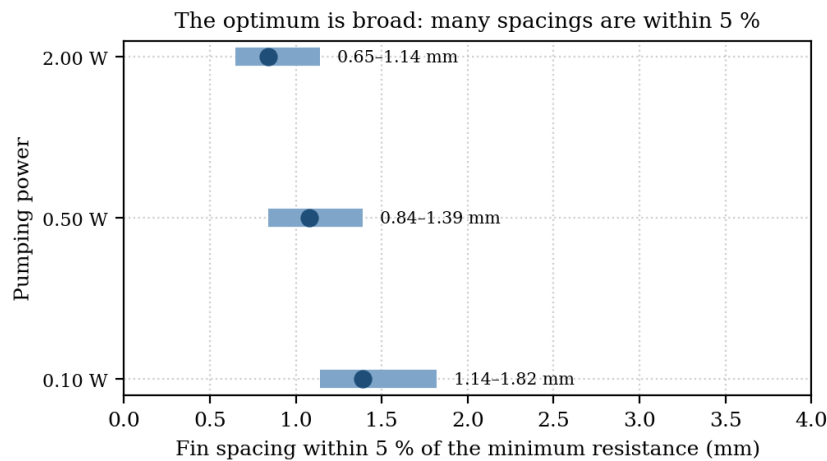


Fig.4: Range of fin spacing within 5 % of the minimum resistance at three power budgets.

At every operating point roughly ten to fourteen distinct fin counts lie within 5 % of the minimum. At 0.50 W any spacing from 0.84 to 1.39 mm — a range of two thirds of the optimum value — performs within 5 %.

The practical reading is that fin spacing should be selected from the acceptable band on manufacturing grounds rather than driven to a computed optimum [15]. Extrusion die life, minimum machinable gap, tolerance on fin thickness, contamination and cleaning access, and the increase in blockage risk at narrow spacing are all real constraints, and within this band they cost almost nothing thermally.

The flatness also bounds how much modelling precision is worth. The correlations used here carry uncertainty of at least several per cent in Nusselt number and friction factor [16], comparable to the width of the 5 % band. Refining the heat transfer model to distinguish designs within that band is therefore not meaningful without experimental calibration.

3.5 Convective against Caloric Resistance

Table 6 and Figure 5 decompose the minimum resistance into its convective and caloric contributions.

Table 6: Decomposition of the minimum resistance.

Power budget	Total R (K/W)	Convective (K/W)	Caloric (K/W)	Caloric share
0.10 W	0.2204	0.0777	0.1427	64.7 %
0.50 W	0.1453	0.0567	0.0886	61.0 %
2.00 W	0.1033	0.0426	0.0607	58.8 %

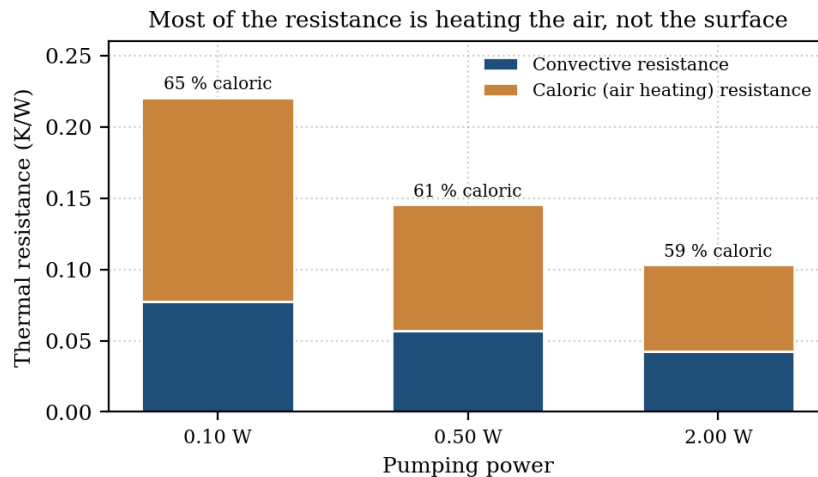


Fig.5: Convective and caloric contributions to minimum thermal resistance.

At every operating point the caloric term is the larger, contributing between 58.8 % and 64.7 % of the total. Most of the temperature rise between the base and the incoming air is not the difficulty of getting heat into the air but the fact that the air gets hot.

This is the finding that explains the previous three. The caloric term depends only on mass flow — no amount of surface area, fin efficiency or spacing optimisation touches it. Adding fins reduces the smaller of the two contributions while raising pressure drop and therefore reducing the mass flow that governs the larger. That is why the optimum is broad, why it moves with the power budget, and why more air outperforms better geometry by an order of magnitude.

It also identifies where a different design would help. Increasing the flow length adds surface but increases air temperature rise along the sink; increasing the frontal area at the same velocity increases mass flow and attacks the dominant term directly. On these results a wider, shorter sink at the same volume would outperform a narrower, longer one, and that is a layout decision made long before fin spacing is chosen.

4. DISCUSSION

4.1 Principal Findings

Four results follow. The optimum fin spacing moves from 1.39 to 0.84 mm as the pumping power rises from 0.10 to 2.00 W, so a single optimum spacing does not exist independently of the air supply. Optimising at a fixed velocity and then operating under a power constraint costs between 0.6 % and 6.7 %. Raising the power budget twentyfold reduced minimum resistance by 53 %, some eight times the largest geometric penalty. And the caloric resistance exceeded the convective resistance at every operating point.

4.2 Design Implications

Table 7 summarises each finding, its implication and the corresponding design rule.

Table 7: Implications.

Finding	Implication	Design rule
Optimum moves 1.39 → 0.84 mm with power	There is no operating-point-free optimum spacing	Fix the air supply before optimising the fin field
Wrong formulation costs 0.6–6.7 %	The error is real but small	Do not spend long on the formulation; spend it on the fan
Power budget worth 53 %	The air path dominates	Attack system pressure drop and fan selection first
Ten to fourteen designs within 5 %	The optimum is broad	Select spacing within the band on manufacturability
Band width comparable to correlation uncertainty	Extra modelling precision is not meaningful	Do not refine the thermal model to resolve designs inside the band
Caloric share 59–65 %	Surface area is not the limit; mass flow is	Increase frontal area rather than fin count where volume allows
Fan curve set its own 0.296 W operating point	The delivered power is an output, not a choice	Solve the fan–sink intersection rather than assuming a flow

The sixth row is the one that would change a layout rather than a detail. If roughly three fifths of the resistance is air heating, then the design variable with the greatest leverage is the mass flow through the sink, which is set by frontal area and velocity rather than by how the frontal area is subdivided. Fin spacing determines how efficiently the available air is used; it does not determine how much air there is.

4.3 Limitations

The model is one-dimensional and treats the base as isothermal, so spreading resistance from a concentrated heat source is omitted entirely [12,13]. For a small die on a large base this can be a substantial additional term and it is insensitive to fin spacing, which would further flatten the optimum and further reduce the relative importance of the fin field — the direction of the omission reinforces the conclusions rather than threatening them.

Bypass flow is not modelled [10,11]. In a real enclosure some air escapes around rather than through the fin field, and bypass increases as the fin field becomes more restrictive, which penalises narrow spacing more than the present model does and would move the optima toward wider spacing. Flow maldistribution across the fin field, entrance and exit losses, and fin tip clearance are likewise absent, and all three act in the same direction.

Correlations for developing laminar flow in rectangular channels carry uncertainty of several per cent, comparable to the width of the 5 % band identified in Section 3.4, so the model cannot discriminate between designs inside that band; this is noted in the results rather than treated as a defect. Radiation is neglected, which is reasonable at these temperature rises for a bare aluminium surface but not for an anodised sink in natural convection. Air properties are evaluated at a single temperature rather than varying along the flow path.

Finally, the geometry, material and dissipation are a single case. Absolute resistances apply only to it, and the balance between convective and caloric terms depends on the ratio of surface area to flow rate, which changes with sink volume and aspect ratio. The qualitative findings — operating-point dependence, optimum flatness and caloric dominance — follow from the structure of Equation (3) and would be expected to hold broadly, but the specific percentages should be recomputed for any real design.

5. CONCLUSION

The optimum fin spacing for a plate-fin heat sink is a function of the air supply, moving from 1.39 mm at a pumping power of 0.10 W to 0.84 mm at 2.00 W. Optimising instead at a nominal velocity costs between 0.6 % and 6.7 % of thermal resistance — a real error, and a small one against the 53 % improvement obtained by raising the power budget over the same range.

Two conclusions follow for practice. Fin spacing is a forgiving variable: ten to fourteen distinct fin counts lay within 5 % of the minimum at every operating point, a band wider than the uncertainty of the correlations used to find it, so spacing should be chosen within that band on manufacturing grounds rather than driven to a computed optimum. And the reason it is forgiving is that between 59 % and 65 % of the total resistance is the heating of the air, a term that no amount of surface area can reduce. The leverage in air cooling lies in how much air reaches the sink, not in how finely the sink divides it.

DECLARATIONS

Ethical approval. Not applicable. This is a computational study; no human or animal subjects were involved.

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Conflicts of interest. The author declares no conflicts of interest, financial or non-financial, in relation to the subject matter of this manuscript.

Product disclaimer. No commercial heat sink, fan or thermal design package is evaluated. Geometry, material and fan characteristic are representative assumptions; absolute thermal resistances apply only to the case modelled and must not be used as design data.

Model disclaimer. Results are obtained from one-dimensional correlations, not from computational fluid dynamics or experiment. Spreading resistance, bypass flow, maldistribution and radiation are omitted, as set out in Section 4.3.

Data and code availability. This is a computational study; no experimental or external datasets were used. All results were generated by the model described in Section 2 using the geometry and properties listed in Table 2, and every reported value can be reproduced from them. The code is available from the corresponding author upon reasonable request.

Author's contribution. Ma'in Abu-Shaikh is the sole author of this work and was responsible for the conception and design of the study, development of the thermal-hydraulic model, analysis and interpretation of the results, preparation of the figures and tables, and drafting, revision and final approval of the manuscript. The author is accountable for all aspects of the work.

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